

CONTENTS

Preface

vii

CHAPTER 1

THE LINEAR PROGRAMMING CONCEPT

1-1. Introduction	1
1-2. The Programming Problem	1
1-3. Linear Programming Defined	6
1-4. Classification of Programming Problems	7
1-5. Mathematical Programming and Automation	10

CHAPTER 2

ORIGINS AND INFLUENCES

2-1. World War II Influences	12
2-2. Economic Models and Linear Programming	16
2-3. Mathematical Origins and Developments	20
2-4. Industrial Applications of Linear Programming	28

CHAPTER 3

FORMULATING A LINEAR PROGRAMMING MODEL

3-1. Basic Concepts	32
3-2. Building the Model	34
3-3. A Transportation Problem	35
3-4. Examples of Blending	42
3-5. A Product Mix Problem	50
3-6. A Simple Warehouse Problem	55
3-7. On-the-job Training	57
3-8. The Central Mathematical Problem	60
3-9. Problems	62

CHAPTER 4

LINEAR EQUATION AND INEQUALITY SYSTEMS

4-1. Systems of Equations with the Same Solution Set	69
4-2. Canonical Systems	75
4-3. Linear Inequalities	81
4-4. Fourier-Motzkin Elimination Method	84
4-5. Linear Programs in Inequality Form	85
4-6. Problems	89

CONTENTS

CHAPTER 5

THE SIMPLEX METHOD

5-1. Simplex Algorithm	94
5-2. The Two Phases of the Simplex Method	100
5-3. Problems	111

CHAPTER 6

PROOF OF THE SIMPLEX ALGORITHM AND THE DUALITY THEOREM

6-1. Inductive Proof of the Simplex Algorithm	120
6-2. Equivalent Dual Forms	123
6-3. Proof of the Duality Theorem	128
6-4. Basic Theorems on Duality	134
6-5. Lagrange Multipliers	140
6-6. Problems	144

CHAPTER 7

THE GEOMETRY OF LINEAR PROGRAMS

7-1. Convex Regions	147
7-2. The Simplex Method Viewed as the Steepest Descent Along Edges	156
7-3. The Simplex Interpretation of the Simplex Method	160
7-4. Problems	166

CHAPTER 8

PIVOTING, VECTOR SPACES, MATRICES, AND INVERSES

8-1. Pivot Theory	173
8-2. Vector Spaces	177
8-3. Matrices	183
8-4. Inverse of a Matrix	189
8-5. The Simplex Algorithm in Matrix Form	195
8-6. Problems	202

CHAPTER 9

THE SIMPLEX METHOD USING MULTIPLIERS

9-1. An Illustration Using Multipliers	211
9-2. The General Method Using Multipliers	217
9-3. Computational Rules Using Multipliers	221
9-4. Problems	226

CONTENTS

CHAPTER 10

**FINITENESS OF THE SIMPLEX METHOD UNDER
PERTURBATION**

10-1. The Possibility of Circling in the Simplex Algorithm	228
10-2. Perturbing Constants To Avoid Degeneracy	231
10-3. Problems	237

CHAPTER 11

VARIANTS OF THE SIMPLEX ALGORITHM

11-1. Complementary Primal and Dual Bases	241
11-2. The Dual Simplex Method	243
11-3. A Self-Dual Parametric Algorithm	245
11-4. The Primal-Dual Algorithm	247
11-5. An Alternative Criterion for Phase I	252
11-6. Problems	253

CHAPTER 12

THE PRICE CONCEPT IN LINEAR PROGRAMMING

12-1. The Price Mechanism of the Simplex Method	254
12-2. Examples of Dual Problems	260
12-3. The Sign Convention on Prices	264
12-4. Sensitivity Analysis Illustrated	265
12-5. Problems	275

CHAPTER 13

GAMES AND LINEAR PROGRAMS

13-1. Matrix Games	277
13-2. Equivalence of Matrix Games and Linear Programs; The Minimax Theorem	286
13-3. Constructive Solution to a Matrix Game (Alternative Proof of Minimax Theorem)	291
13-4. Problems	297

CHAPTER 14

THE CLASSICAL TRANSPORTATION PROBLEM

14-1. Historical Summary	299
14-2. Elementary Transportation Theory	300
14-3. Computational Algorithm for the Transportation Problem	308
14-4. Problems	314

CONTENTS

CHAPTER 15

**OPTIMAL ASSIGNMENT AND OTHER DISTRIBUTION
PROBLEMS**

15-1. The Optimal Assignment Problem	316
15-2. Allocation with Surplus and Deficit	322
15-3. Fixed Values and Inadmissible Squares	330
15-4. Problems	332

CHAPTER 16

THE TRANSSHIPMENT PROBLEM

16-1. Equivalent Formulations of the Model	335
16-2. The Equivalence of Transportation and Transshipment Problems	342
16-3. Solving a Transshipment Problem by the Simplex Method	346
16-4. Problems	351

CHAPTER 17

NETWORKS AND THE TRANSSHIPMENT PROBLEM

17-1. Graphs and Trees	352
17-2. Interpreting the Simplex Method on the Network	357
17-3. The Shortest Route Problem	361
17-4. Problems	366

CHAPTER 18

VARIABLES WITH UPPER BOUNDS

18-1. The General Case	368
18-2. The Bounded Variable Transportation Problem and Generalizations	377
18-3. Problems	383

CHAPTER 19

MAXIMAL FLOWS IN NETWORKS

19-1. Ford-Fulkerson Theory	385
19-2. The Tree Method for Solving Maximal Flow Problems	398
19-3. Problems	403

CHAPTER 20

**THE PRIMAL-DUAL METHOD FOR TRANSPORTATION
PROBLEMS**

20-1. Introduction	404
20-2. The Ford-Fulkerson Algorithm	405
20-3. Problems	411

CONTENTS

CHAPTER 21

THE WEIGHTED DISTRIBUTION PROBLEM

21-1.	The Near-Triangularity of the Basis	413
21-2.	Linear Graph Structure of the Basis	420
21-3.	A Subclass with Triangular Optimum Bases	424
21-4.	Problems	431

CHAPTER 22

PROGRAMS WITH VARIABLE COEFFICIENTS

22-1.	Wolfe's Generalized Program	433
22-2.	Notes on Special Cases	440
22-3.	Problems	446

CHAPTER 23

A DECOMPOSITION PRINCIPLE FOR LINEAR PROGRAMS

23-1.	The General Principle	448
23-2.	Decomposition Principle, Animated	455
23-3.	Central Planning without Complete Information at the Center	462
23-4.	Decomposing Multi-stage Programs	466
23-5.	Problems	469

CHAPTER 24

CONVEX PROGRAMMING

24-1.	General Theory	471
24-2.	Homogeneous Objectives and the Chemical Equilibrium Problem	479
24-3.	Separable Convex Objectives	482
24-4.	Quadratic Programming	490
24-5.	Problems	497

CHAPTER 25

UNCERTAINTY

25-1.	Scheduling To Meet Variable Cost	499
25-2.	Scheduling To Meet an Uncertain Demand	503
25-3.	On Multi-stage Problems	507
25-4.	Problems	511

CHAPTER 26

DISCRETE VARIABLE EXTREMUM PROBLEMS

26-1.	Survey of Methods	514
26-2.	Gomory's Method of Integer Forms	521
26-3.	On the Significance of Solving Linear Programming Problems with Some Integer Variables	535

CONTENTS

CHAPTER 27

**STIGLER'S NUTRITION MODEL: AN EXAMPLE OF
FORMULATION AND SOLUTION**

27-1. Problems in Formulating a Model	551
27-2. Numerical Solution of the Nutrition Problem	557
27-3. Problems	566

CHAPTER 28

**THE ALLOCATION OF AIRCRAFT TO ROUTES
UNDER UNCERTAIN DEMAND**

28-1. Statement and Formulation	568
28-2. Numerical Solution of the Routing Problem	580

BIBLIOGRAPHY	592
---------------------	------------

References to the Bibliography are given in text and at the end of each chapter (see in particular the end of Chapter 2).

INDEX	617
--------------	------------