Contents

Chapter 1 Introduction 1

1.1 Problem Statement and Basic Definitions 2
1.2 Illustrative Examples 4
1.3 Guidelines for Model Construction 26
Exercises 30
Notes and References 34

Part 1 Convex Analysis 37

Chapter 2 Convex Sets 39

2.1 Convex Hulls 40
2.2 Closure and Interior of a Set 45
2.3 Weierstrass's Theorem 48
2.4 Separation and Support of Sets 50
2.5 Convex Cones and Polarity 62
2.6 Polyhedral Sets, Extreme Points, and Extreme Directions 64
2.7 Linear Programming and the Simplex Method 75
Exercises 86
Notes and References 93

Chapter 3 Convex Functions and Generalizations 97

3.1 Definitions and Basic Properties 98
3.2 Subgradients of Convex Functions 103
3.3 Differentiable Convex Functions 109
3.4 Minima and Maxima of Convex Functions 123
3.5 Generalizations of Convex Functions 134
Exercises 147
Notes and References 159

Part 2 Optimality Conditions and Duality 163

Chapter 4 The Fritz John and Karush–Kuhn–Tucker Optimality Conditions 165

4.1 Unconstrained Problems 166
4.2 Problems Having Inequality Constraints 174
4.3 Problems Having Inequality and Equality Constraints 197
4.4 Second-Order Necessary and Sufficient Optimality Conditions for Constrained Problems 211
Exercises 220
Notes and References 235

Chapter 5 Constraint Qualifications 237

5.1 Cone of Tangents 237
5.2 Other Constraint Qualifications 241
5.3 Problems Having Inequality and Equality Constraints 245
Exercises 250
Notes and References 256

Chapter 6 Lagrangian Duality and Saddle Point Optimality Conditions 257

6.1 Lagrangian Dual Problem 258
6.2 Duality Theorems and Saddle Point Optimality Conditions 263
6.3 Properties of the Dual Function 276
6.4 Formulating and Solving the Dual Problem 286
6.5 Getting the Primal Solution 293
6.6 Linear and Quadratic Programs 298
Exercises 300
Notes and References 313

Part 3 Algorithms and Their Convergence 315

Chapter 7 The Concept of an Algorithm 317

7.1 Algorithms and Algorithmic Maps 317
7.2 Closed Maps and Convergence 319
7.3 Composition of Mappings 324
7.4 Comparison Among Algorithms 329
Exercises 332
Notes and References 340

Chapter 8 Unconstrained Optimization 343

8.1 Line Search Without Using Derivatives 344
8.2 Line Search Using Derivatives 356
8.3 Some Practical Line Search Methods 360
8.4 Closedness of the Line Search Algorithmic Map 363
8.5 Multidimensional Search Without Using Derivatives 365
8.6 Multidimensional Search Using Derivatives 384
8.7 Modification of Newton's Method: Levenberg–Marquardt and Trust Region Methods 398
8.8 Methods Using Conjugate Directions: Quasi-Newton and Conjugate Gradient Methods 402
8.9 Subgradient Optimization Methods 435
Exercises 444
Notes and References 462

Chapter 9 Penalty and Barrier Functions 469

9.1 Concept of Penalty Functions 470
9.2 Exterior Penalty Function Methods 475
9.3 Exact Absolute Value and Augmented Lagrangian Penalty Methods 485
9.4 Barrier Function Methods 501
9.5 Polynomial-Time Interior Point Algorithms for Linear Programming Based on a Barrier Function 509
Exercises 520
Notes and References 533

Chapter 10 Methods of Feasible Directions 537

10.1 Method of Zoutendijk 538
10.2 Convergence Analysis of the Method of Zoutendijk 557
10.3 Successive Linear Programming Approach 568
10.4 Successive Quadratic Programming or Projected Lagrangian Approach 576
10.5 Gradient Projection Method of Rosen 589

10.6 Reduced Gradient Method of Wolfe and Generalized Reduced Gradient Method 602
10.7 Convex–Simplex Method of Zangwill 613
10.8 Effective First- and Second-Order Variants of the Reduced Gradient Method 620
Exercises 625
Notes and References 649

Chapter 11 Linear Complementary Problem, and Quadratic, Separable, Fractional, and Geometric Programming 655

11.1 Linear Complementary Problem 656
11.2 Convex and Nonconvex Quadratic Programming: Global Optimization Approaches 667
11.3 Separable Programming 684
11.4 Linear Fractional Programming 703
11.5 Geometric Programming 712
Exercises 722
Notes and References 745

Appendix A Mathematical Review 751

Appendix B Summary of Convexity, Optimality Conditions, and Duality 765

Bibliography 779

Index 843